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APPLICATION OF A THEOREM ON THE ZEROS OF STOCHASTIC FUNCTIONS TO TURBULENCE MEASUREMENTS

H. W. Liepmann

Translation of: "Die Anwendung eines Satzes ueber die Nullstellen Stochastischer Funktionen auf Turbulenzmessungen," Helvetica Physica Acta, Volume 22, No. 2, March 18, 1949, pp. 119-126

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16. Abstract

The mean number of null-positions of a stochastic function, according to Rice, is related directly to the mean square of their derivative. If one uses this theorem on the components of the fluctuatio fluctuation rate in isotropic turbulence, then using certain assumptions, he can determine the length λ characteristic for the dissipation by counting the number of zero-positions. Measurements of this kind have been made, and the results were compared with three other independent determinations.

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APPLICATION OF A THEOREM ON THE ZEROS OF STOCHASTIC FUNCTIONS TO TURBULENCE MEASUREMENTS

H. W. Liepmann[1]

1. The Number of Null-Positions of a Stochastic Function /119*

Let J(t) be a stochastic function of time t. The temporal mean value $\overline{J(t)}$ may be zero and the process described by J(t) stationary, i.e., the correlation function

$$\psi(\tau) = \overline{J(t)\,J\,(t+\tau)}$$

depends only on τ and not on t. How often does the function J(t) assume the value ξ on the average in the time unit? Questions of this kind have been discussed by Rice [1][2].

Under certain suppositions ($\xi = 0$) is valid for the special mean number of zero-positions N_0, in the unit of time

$$N_0 = \frac{1}{\pi}\sqrt{-\frac{\psi''(0)}{\psi(0)}} \tag{1}$$

Since for a stationary process with $\psi(\tau)$ is valid:

$$-\psi''(0) = \overline{\left(\frac{dJ}{dt}\right)^2} = \overline{J'^2}$$

it follows

$$N_0^2 = \frac{\overline{J'^2}}{\pi^2 \overline{J^2}} \tag{2}$$

and thus one has a relation between the mean number of zero-positions and the mean square of the derivative. A spectral density F(n) is defined in such a way that /120

$$\psi(0) = \overline{J^2} = \int_0^\infty F(n)\,dn$$

then

$$\psi(\tau) = \int_0^\infty F(n)\cos(2\pi n\tau)\,dn$$

is valid,

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*Numbers in the margin indicate pagination in the foreign text.
[2]Additional references are cited in Rice's work.

and

$$- \psi''(0) = 4\pi^2 \int_0^\infty n^2 F(n)\, dn. \tag{3}$$

Then it follows also from (1) that:

$$N_0^2 = 4 \frac{\int_0^\infty n^2 F(n)\, dn}{\int_0^\infty F(n)\, dn} = 4\,\overline{n^2}.$$

The relation (3) is analogous to the simple harmonic oscillation, for which $N_0^2 = 4n^2$.

The assumptions which lead to the relationship (1) could be indicated by a simple consideration. For a stricter derivation, see Rice (loc. cit).

Let $p(\xi)d\xi$ be the probability of finding $J(t)$ at time t in the interval ξ, $\xi + d\xi$. Now if one does not ask about the duration of the interval ξ, $\xi + d\xi$, but only about the number of transits, then it obviously is immaterial how long the function remains through one transit in the interval. In order to find the probability of one passage, so as to find the probability $J(t)$ in the interval ξ, $\xi + d\xi$, at any given time one must therefore divide by the time T which $J(t)$ spends during this passage in $d\xi$. The relation between the number of transits and the mean square of the derivative $\overline{J'(t)^2}$ then naturally occurs such that the time T depends on $J'(t)$. Let $p(\xi, \eta)d\xi d\eta$ be the probability that $J(t)$ is in the interval ξ, $\xi + d\xi$ and simultaneously $J'(t)$ is in the interval η, $\eta + d\eta$. Then it is probable that $J(t)$ exceeds the interval with certain derivative η

$$\frac{p(\xi, \eta)\, d\xi\, d\eta}{T} = p(\xi, \eta)\, |\eta|\, d\eta$$

since

$$T = \frac{d\xi}{|\eta|}$$

Furthermore if one integrates η over all possibilities, one obtains the number of transits by ξ per time unit: /121

$$N_\xi = \int_{-\infty}^{+\infty} |\eta|\, p(\xi, \eta)\, d\eta \tag{4}$$

From equation (4), (1) follows directly under the assumption, that $p(\xi, \eta)$ is a Gaussian distribution and that this is process stationary, i.e., $\overline{J(t)J'(t)} = 0$.

It is essential to note that J(t) and J'(t) are statistically independent, that is, that $p(\xi, \eta)$ is a product of a function of ξ and a function of η. If one writes $p(\xi, \eta) = l(\xi)\, m(\eta)$, then from (4) it follows that

$$N_\xi = l(\xi) \int_{-\infty}^{+\infty} |\eta|\, m(\eta)\, d\eta = l(\xi)\, \overline{|\eta|} \tag{5}$$

For a Gaussian distribution the disappearance of the correlation JJ' suffices. In general JJ' = 0 indeed is necessary, but not sufficient for the statistical independence of J(t) and J'(t).

For linear processes one can show in general that J(t) and J'(t) have Gaussian distribution. For the radiation of a black body, one finds such evidence in von Laue [2] and the corresponding acoustical and electrical case is found, e.g., in Rice's cited work. The possibility of equation (1) being used in turbulence measurements to my knowledge, was first mentioned by Dryden.

Turbulent fluctuations however obey a nonlinear equation, and consequently cannot be included in a Gaussian distribution in the same way, as e.g., in radiation theory. On the contrary, the derivative of a turbulent rate component certainly is not disturbed in a Gaussian way. That follows essentially from the motion equations in the form given to them by Karman and Howarth [3] and has been confirmed by Townsend's [4] measurements. Yet it is interesting to test how far this — in general weak — deviation from a Gaussian distribution operates in the number of zero-positions and how widely the number of zero-positions characterizes a turbulent fluctuation field.

From a technical measurement point of view, it is a very attractive method since, with a counting arrangement, direct averages can be taken over almost arbitrarily long times.

2. Relations to Isotropic Turbulence /122

Under isotropic turbulence, as is known from G. I. Taylor, one understands a fluctuation field, in which functions of the velocity components $u_i(x_k, t)$ and their derivatives are invariant as regards the translation, rotation and reflection of the coordinate system. In experiments one produces such a field by letting as uniform and turbulence-free a stream as possible flow through a grating at constant speed U. The field in the wake of the grating is then nearly isotropic and homogeneous in miniature. The components of the variation rate at a fixed point $u_i(t)$ can then be identified with the stochastic function J(t). For technical measurement it is simplest to investigate the component $u_i(t)$ in the direction of the mean velocity U.

The result for the mean square of the derivative $\overline{\left(\frac{du_1}{dt}\right)^2}$ that was obtained from counting the zero-positions, could be compared with the results of the two following independent methods: namely, first with the direct measurement of $\overline{\left(\frac{du_1}{dt}\right)^2}$ by differentiation by means of a capacity-resistance arrangement (cf. Townsend [4]) and also with measurement of the spectral density F(n) from the $\overline{\left(\frac{du_1}{dt}\right)^2}$ by graphic interpretation of $\int_0^\infty n^2F(n)dn$ corresponding equation (3) can be acquired.

So far we have considered only time functions. For the dynamics of turbulence, however, the spatial correlation functions are especially important. These spatial correlation functions according to V. Karman are best introduced in the form of correlation tensors.

In isotropic, homogeneous turbulence then, a scalar function, e.g., f(x), determines the correlation tensor of the second order:

$$R_{ij} = \frac{\overline{u_i(x_k, t)\, u_j(x_k', t)}}{\overline{u^2}} .$$

The relation between the time correlation function $\psi(\tau)$ and f(x) in general is obtained by setting:

$$\tau \longrightarrow \frac{x}{U}$$
$$\frac{d}{dt} \longrightarrow -U\frac{d}{dx} \qquad (6)$$

i.e., it is assumed, that the turbulence is transported undistorted with the speed U. This substitution — even for small fluctuations — to my knowledge never has been proved to be theoretically flawless, and it can rightly be doubted [5]. The few measurements that permit a comparison between space and time correlation, admittedly for the present have shown no systematic differences, and the same holds also for measurements in this work. If one assumes meanwhile that the (6) substitutions are allowed, then one can write: /123

$$\frac{1}{\overline{u_1^2}}\overline{\left(\frac{du_1}{dt}\right)^2} = \frac{U^2}{\overline{u_1^2}}\overline{\left(\frac{du_1}{dx}\right)^2} = -U^2 f''(0) \equiv \frac{U^2}{\lambda^2}$$

λ is a quantity which is characteristic for the dissipation of the turbulent energy. The dissipation equation in isotropic turbulence, as is known, runs (see e.g., [3])

$$-\frac{3}{2}\varrho U\frac{d\bar{u}_1^2}{dx} = \frac{15\mu u_1^2}{\lambda^2} \tag{7}$$

and hence the quantity λ can be determined by measurements of turbulence fading. Such measurements also were carried out, and the values of λ so obtained could be compared with the mentioned measurements of $\overline{\left(\frac{du_1}{dt}\right)^2}$. In the following — with the reservation mentioned — also for the quantity $\frac{1}{U^2\bar{u}_1^2}\overline{\left(\frac{du_1}{dt}\right)^2}$ the customary short $\frac{1}{\lambda^2}$ has been written.

3. Measuring Apparatus

Windtunnel

The measurements were conducted in a small duct with a cross-section of 20"•20". The degree of turbulence of this duct without grating is small:

$$\left[\frac{\bar{u}_1^2}{U^2}\right]^{1/2} \cong 3\cdot 10^{-4}$$

A grating of 1.27 cm mesh width was used to produce isotropic turbulence. The measurements were done with flow rates of 630 and 1130 cm/sec.

Hot Wire Arrangement

The fluctuations of the flow rate were absorbed with help of hot wires and compensated amplification. Wollaston wire with a $1.25\cdot 10^{-4}$ cm thick platinum core was used. Wires were 1-2 cm long. The platinum wire, after dissolving the /124 silver coating, was soldered on the tips of fine sewing needles. The arrangement is true in frequency within 2% between 2 and 10,000 Hz.

Counting Device

The zero-positions of the amplifier current were taken up with a photomultiplier cell onto the screen of a cathode ray oscillograph and by means of a gear reducer every 2^9 = 512th zero-position counted.

Differentiation

It was possible to differentiate the anplifier current by means of a capacity-resistance system similar to that of Townsend [4]. The degree of amplification here is proportional to the frequency between 2 and 10^4 Hz.

Measurement of the Spectral Density

The spectral distribution of the amplifier current could be measured with aid of a Hewlett-Packer wave analyzer with constant band width.

Corrections

The results have to be corrected for the full length of the hot wire. For that it means that not $u_1(t)$ is measured, but

$$u_1(t,l) = \frac{1}{l}\int_0^l u_1(t,y)\,dy$$

where l is the length of the hot wire. For isotropic turbulence the correction formulas can be specified. For the intensity measurements, one has

$$\overline{u^2} = \overline{u^2}\text{ measured}\left[1+\frac{1}{6}\left(\frac{l}{\lambda}\right)^2\right],$$

for the λ measurements

$$\lambda^2 = \lambda^2\text{ measured}\left[1-\frac{G-3}{18}\left(\frac{l}{\lambda}\right)^2\right] \quad \text{mit} \quad G=\lambda^4 f^{\mathrm{IV}}(0).$$

It is clear that the correction formulas make sense only when the corrections are small, so that one comes out with relatively crude values for λ and G in correction terms. With the measurements in this work $l/\lambda \leq 0.5$; G was estimated from the spectrum: $G \cong 10\text{-}13$. In addition the corrections were determined experimentally by use of wires of various lengths. For the higher velocity /125
(1130 cm/sec), it gave: $(G-3)/18 = 0.46$. This value agrees closely enough with the measurements of G.

The counting device moreover must be corrected for the ultimate resolution power. At present this correction seems to be the least certain and therefore contributes to the fact that not the deviations in the results of this counting method from the other methods can be regarded as real. The probability of having two null-positions tightly in succession can be stated. A suitable formula is found in Rice (loc. cit.). The medial number N_D of dual zero-positions within the small time T is given by

$$N_D = \frac{\pi^2}{16} N_0[G-1](N_0T)^2.$$

If one identifies T with the resolution process, then one obtains a correction formula for the measured number of zero-positions. The determination of T causes difficulties. The previous determinations yielded for the arrangement $T \cong 10^{-4}$ sec. So the correction is small. But the value is not too reliable and certainly must be delineated better. Besides, still another correction is added, in that on occasion very rapid transits through zero do not get counted. In this respect surely the arrangement still is improvable.

4. Results

λ or better λ^2 was measured by the different methods at speeds of 630 and 1130 cm/sec at a distance of 81 mesh-widths behind the screen. The results are summarized in the table.

TABLE

Methods	$\lambda^2 \cdot 10^2$ cm^2			
	Zero-positions	Spectrum	Differentiation	Dissipation
1130 cm/sec	17.3	15.3	12.9	13.9
630 cm/sec	23.9	25.8	18.7	22.4

The measurements are scarcely more precise than ±10% in λ^2. More exact limits of error cannot be stated at present. One finds namely that the measured values can be periodically reproduced more exactly, but then often are changed systematically by several percent. How far such changes are occasioned by the apparatus, e.g., by irregularities in the grating, still has to be checked. /126

Within these limits of error, as is evident from the table, no systematic difference between the results of the various measuring methods can be detected for sure. For the higher velocity the counting method indeed gives too large a value for λ^2, but not for the lower velocity. This result makes one suspect that the resolution power of the arrangement has not yet been considered sufficiently.

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Neither the influence of a deviation from Gaussian distribution, nor a difference between time and space differentiation can therefore be detected for sure within these limits of error.

At the California Institute of Technology at present, a detailed investigation of isotropic turbulence is in progress for the National Advisory Committee for Aeronautics. The question discussed here came out in the scope of this work.

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